

### Faculty Member Contact Information

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| <b>Name</b>         | Jireh Loreaux, PhD         |
| <b>Contact Info</b> |                            |
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| <b>Department</b>   | Mathematics and Statistics |

### 1 Funded, 0-1 Unfunded URCA Assistant(s)

**Are you willing to work with students from outside of your discipline? If yes, which other disciplines?**

- No

**How many hours per week will your student(s) be required to work in this position?**  
(Minimum is 6 hours per week; typical is 9)

- 9

**Will it be possible for your student(s) to earn course credit?**

- No

**Location of research/creative activities:**

- anywhere, although there would be regular meetings with me in my office SE 2354

**Brief description of the nature of the research/creative activity?**

- I am writing a book about filters. A filter is an abstract mathematical object which allows one to unify many concepts of limits simultaneously. Filters are standard tools in the practice of the formalization of mathematics, but are often considered a rather niche topic within most other areas of mathematics (except in logic or order theory, where they are common for other reasons), even within analysis and topology.
- The purpose of this book is to serve as an introduction to filters for both students and research mathematicians alike. Its goal is to present them in a way which provides some motivation for their development, as well as to teach the reader to develop a strong intuition for working with filters. The framework the book adopts in this regard is that filters should be conceptualized as a kind of "generalized set". It proceeds to develop the theory, but firmly root in the analogy with sets. Moreover, it motivates why it is

necessary to have these generalized sets, and why other approaches for dealing with limits are suboptimal in comparison.

- The book will be written with a focus on integration with the Lean theorem prover and programming language, as well as with its mathematical library, Mathlib. All the exercises, and much of the exposition, will be presented with reference to Lean. This text will assume *some* prior exposure to Lean to maximize reader understanding, but hopefully not so much as to be burdensome. Nevertheless there will be a preface with a brief introduction to reading and writing Lean.

### **Brief description of student responsibilities?**

- Students engaged in the URCA will *not* be expected to have any prior knowledge of or experience with filters, nor Lean. However, a willingness to learn about both is a necessity. The activities of the student fall into primarily three categories:
  1. Writing solutions in Lean to exercises in the book.
  2. Reading and comprehending the book, and pointing out areas they found hard to comprehend, and why.
  3. Developing new exercises for the book to elucidate new connections they found in the process of learning that were not otherwise described.

**URCA Assistant positions are designed to provide students with *research or creative activities* experience. As such, there should be measurable, appropriate outcome goals. What exactly should your student(s) have learned by the end of this experience?**

- By the end of this experience, students participating in this URCA should have:
  1. learned the fundamentals of the Lean theorem prover and how to use it to formally verify mathematical results.
  2. developed a strong intuition for filters and how they arise in practice as a way to describe limits in mathematics.
  3. learned about the process of writing a book, and how editing a text takes time, care and strong attention to detail, especially as regards the intended readers' mindset and expertise.

The outcomes (1) and (2) are measurable in the form of: "how many exercises were students able to successfully formalize?" Outcome (3) is less easily measured quantitatively, but I think it is a worthwhile outcome nonetheless.

### **Requirements of Students**

**If the position(s) require students to be available at certain times each week (as opposed to them being able to set their own hours) please indicate all required days and times:**

- No days are required, I can be flexible about meeting times.

**If the location of the research/creative activities involves off campus work, must students provide their own transportation?**

- N/A

**Must students have taken any prerequisite classes? Please list classes and preferred grades:**

- Required: Math-350: Introduction to Analysis

**Other requirements or notes to applicants:**

- Students should have access to a computer with at least 8 GB of RAM (but preferably 16 GB or more).